Where does the calculus go? A follow up investigation of how calculus ideas are used in core engineering coursework

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***Abstract*—Mathematics courses are a major source of engineering student attrition. Many engineering students drop out before even taking an engineering course. While the mismatch between mathematics content and following engineering coursework is already a concern, it is often studied heuristically by talking to engineering faculty. Few rigorous studies empirically document when students actually need to use calculus knowledge in their coursework. We mapped how foundational calculus concepts are used to solve assigned problems in topics covered in the widely required Engineering Statics course. To create this mapping, we used the mathematics-in-use technique of Czocher et al. We present preliminary evidence of the match, or mismatch, between what calculus is taught and learned in mathematics courses and what is expected of students in following engineering coursework. For this work-in-progress, we focused on presenting the alignment between calculus concepts/skills and topics taught in Statics.**

I. Introduction

The United States demands more engineering graduates to fill projected industrial job growth [4]. However, since many students drop out of the engineering major, too few engineering graduates join industry [2]. More students drop out of engineering after failing a mathematics course than after failing an engineering course [3]. Most engineering programs require a standard calculus sequence of Calculus I, Calculus II, Calculus III, Linear Algebra, and Differential Equations; students must pass prerequisite mathematics courses from the calculus sequence to continue into core engineering coursework [5]. The strict prerequisite chain can particularly hamper students who are already disadvantaged due to disability or lack of access to high school calculus [6]. Students who do not start college calculus-ready or who fail a course in the calculus sequence often struggle to complete their engineering degrees before running out of financial aid.

Because the calculus sequence has such a strong impact on engineering graduation, we must justify these high-failure prerequisites. How much of the mathematical content of these courses is truly required by core engineering? In the following, we present preliminary results for a study examining the matches and mismatches between the mathematics that is taught in calculus and how that mathematics is applied in core engineering courses.

This study will map the knowledge learned in the prerequisite mathematics courses to when that knowledge is used (if ever) in core engineering courses, following the mathematics-in-use technique developed by Czocher et al [1]. Findings from this study will allow educators to reform mathematics curriculum with confidence so that students' success in subsequent core courses will not be impeded. Some skills/concepts may not appear in subsequent courses and thus could be considered for removal in order to make room for other topics. While this evidence does not single-handedly nominate a technique for removal from the standard curriculum, the burden of proof must fall on the affirmative to keep a topic that lacks application in further coursework. Mathematics-in-use was previously used to study alignment between calculus and differential equations; this study extends the technique to study the alignment of calculus with statics instead.

II. Methods

A. Research Questions

Research Question 1) Which calculus skills/concepts are applied in core engineering courses? Research Question 2) What skills/concepts not currently taught in calculus could be applied to topics in core sophomore engineering courses?

B. Data Selection

We have chosen to study how calculus skills/concepts are applied in Statics and Circuits courses, as they are often the first engineering courses that students take following the calculus sequence and are considered the gateway to upper-level engineering courses. Mechanical, Civil, Electrical, Materials, Nuclear, Aerospace, Agricultural, Industrial and Systems engineering all require at least one of Statics or Circuits. The content of these courses is relatively fixed from institution to institution so the analysis should generalize well [7]. We obtained the homework and quiz problems for statics directly from the instructor for this preliminary analysis. For this

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		Statics Topics																			
		force vectors	unit vectors	force along a line	projections	cross product	Particle Equilibrium	moment of a force	couples	equivalent systems	distributed loading	rigid body equilibrium	2d reactions	3d problems	method of joints	method of sections	frames&machines	internal forces	Shear, bending	relation of w,v,& M	friction
Calculus Concepts	derivative										x								x	x	
	integral										x							x	x	x	
	fundamental theorem							x	x			x						x	x	x	
	limit																				
	approximation																				
	sequences and series																				
	Riemann sums																	x			
	polar and parametric			x								x	x	x							
	continuity			x								x							x		
	optimization																				
Calculus Skills	derivative computations										x								x	x	
	integration techniques										x							x	x	x	
	sequences/series																				
	algebraic expressions						x	x	x	x	x	x			x	x	x	x	x	x	x
	area/volume					x					x							x			
	parametric equations												x	x							
	polar coordinates												x	x							
	trig manipulations	x	x		x	x	x	x					x	x					x	x	
	logs/exp																				
	epsilon-delta																				
	listening/reading Comprehension				x			x			x					x					
	definitions/notation		x		x	x	x		x	x	x	x			x	x					

Fig. 1. A mock-up result matrix. Topics from statics are correlated with required topics and skills in calculus. It is possible that some rows will be empty (red), indicating a calculus skill or topic that does have application in statics. It is possible that a column will be very dense (green), indicating a topic from statics that applies many elements of calculus. NOTE: This table is only a mock-up of what the forthcoming paper will contain, the data in this table is not the result of actual analysis.

work-in-progress, we present initial analysis of the data from one problem from Statics only. The task presented here is a prototypical homework problem from engineering statics. This problem was chosen because it contains substantial calculus content to illustrate the analysis technique, but many other tasks would have been equally suitable.

III. Analysis

We employed mathematics-in-use technique for the analysis of course artifacts [1]. This technique involves solving a problem completely without skipping a single step, especially steps that might be obvious to an expert.

The resulting data is a narrative of the problem solution and a list of the concepts and skills that are needed to solve that problem. A mock-up of such a matrix is shown in Figure 1 Analysis was conducted by a team of two researchers: the first author, a doctoral student in Electrical Engineering and Engineering Education, assisted by a doctoral student in Materials Science with relevant training in mathematics and Statics.

Mathematics-in-use makes an important distinction between topics and skills/concepts [1]. Topics are broad ideas found in tables of contents, syllabi, or at the top of lecture slides (e.g. Bending Moments). Concepts are lower-level ideas about objects (e.g. such as derivatives express a rate of change. Skills are the sequences of steps used to solve a particular type of problem (e.g. how to compute the derivative of a polynomial).

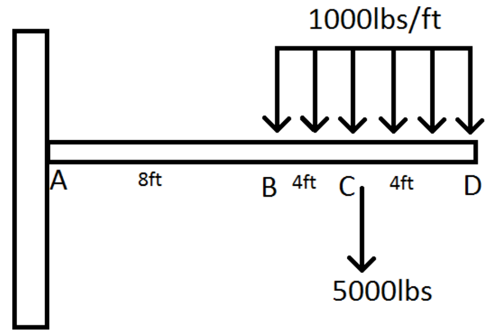

Fig. 2. A typical schematic for a cantilever beam problem.

There is an expected many-to-one mapping from skills/concepts onto topics. A problem often requires many skills/concepts, but covers only a single topic. Data on the alignment between prerequisite skills/concepts and target course topics will be presented in a matrix (see Figure II). This format lays bare the most immediately recognizable content from statics (topics), but shows the base elements from the prerequisite Calculus course required to solve that content. To make the list of topics in Statics, we used the course syllabus and the textbook table of contents from the statics course taught by our Mechanical Engineering Department. The concepts and skills for calculus were taken from the list generated by Czocher [1].

The "mathematics-in-use technique must document all the ways the student could be expected to know how to solve the problem. For example, a truss problem might use the method of sections or method of joints. To increase the likelihood that all solution paths will be followed, two researchers separately derived their own solutions to the problem and coded their solutions as to which skills/concepts were needed to solve the problem. In addition, special attention was paid to possible problem paths precluded by course sequencing. For example, truss problems are usually solved using systems of linear equations rather than with Linear Algebra, despite its usefulness, because it is traditionally taught after statics.

IV. Data

For this work in progress presentation, we have selected one prototypical task from Statics: a shear and bending moment diagram. Consider a 16 ft long horizontal cantilevered beam affixed to a vertical wall shown in Figure 2. A point load of 5000 lbs pulls downward at a point 4 feet from the beams end, and a distributed load of 1000 lbs/ft pushes downward on the beam from the middle to its unsupported end. Students must plot the shear and bending moment diagrams for the beam.

To solve this problem, students must use the concept of mechanical equilibrium, both translational and rotational, to solve for reaction forces (A_x and A_y) and moments (M_{ext}) as well as internal shear force ($V(x)$) and bending moment ($M(x)$). In addition, students must distinguish between the local/internal equilibrium of a point in the beam and the global equilibrium of the entire extended body.

In the first stage of the problem, students must apply the equations of equilibrium (i.e., $\sum F = 0$ and $\sum M = 0$) to find the reaction forces and moment. Students must carefully track the sign conventions in statics, particularly that counterclockwise moments are positive (Skill:definitions/notation). They must know that the total moment of the distributed load can be considered to act at its centroid when treating the beam as a rigid body (though this is not the case when analyzing the shear force). The student must choose a convenient coordinate center to make equations easier to solve. The resulting linear system is uncoupled and only requires arithmetic.

In the next phase of the problem, the students must divide the beam into sections where behavior changes qualitatively. This requires knowledge of piecewise functions and cut placement locations.

In the first segment of the beam (A to B), the sign conventions are needed, both for the direction of the internal reaction shear force and bending moment. In addition, students must recognize that this shear force is a function $V(x)$, not a static number V (likewise for $M(x)$). The variable x is not fixed and represents any choice of position. One cannot solve for xs numerical value; it is a variable, not an unknown. The student must also create internal variables for shear force and bending moment at a point, V and M, which are not included in the initial diagram.

When constructing the equation for the first segment, students must recognize that a constant function is still a function and not a number. Because Bending Moment ($M(x)$) is the integral of shear force, students must then integrate the function $V(x)$ using the rules for polynomials (Skill:integration techniques). The free variable of integration, +C, is resolved through the boundary conditions. The boundary conditions are complicated by the sign and notational conventions regarding the direction of shear force at the joint, where $M(x = A) = -M_A$ (Skill:definitions/notation). In the way the students have been taught the relationship between the shear force and bending moment, it is unlikely they would be able to set up the problem with the boundary condition included in the integration limits. The student might be able to check their work by noticing that the magnitude of the internal forces decrease towards the unsupported end of the beam. The initial condition originates from the forces at the joint. In a second possible approach, a student might also opt to use graphical integration rather than integrating the algebraic expressions for $V(x)$ to obtain $M(x)$.

In the second segment (B to C), the student must construct an expression for the amount of force up to a point (Skill: algebraic expressions). The integration itself requires again only integration of a polynomial. When the +C is evaluated this time, the constraint of physical continuity is applied at point B (Concept: continuity).

The third segment (C to D) includes the point load. The integration of this term again requires only polynomial integration. This time the +C is resolved with the boundary condition not being taken from continuity, but from the zero shear condition of a free unsupported end. To obtain the shear and bending moment diagrams, the students just plot the functions they obtained by integration.

TABLE I
EPISTEMOLOGICAL MISMATCHES. CALCULUS CONTENT HAS DIFFERENT USE IN MATHEMATICS AND ENGINEERING

Topic	In CALCULUS	In STATICS
Integral	Anti-derivative; measurement of area, volume, accumulation	Defined relationship between quantities
Continuity	Property to be checked	Mathematical tool for ensuring desirable properties
Fundamental Theorem of Calculus	Formal justification for anti-derivatives instead of definite integrals, shortcut for computing certain derivatives and integrals	Creates free parameter for satisfying initial boundary conditions

TABLE II
CONCEPTS AND SKILLS USED IN THE STATICS TOPIC: SHEAR AND BENDING MOMENTS.

CONCEPTS		SKILLS	
Derivative		Derivative computations	
Integral		**Integration techniques**	x
Fundamental theorem		Limit calculations	
Limit		Sequences/series	
Approximation		**Algebraic expressions**	x
Riemann sums		Area/volume	
Parametric/polar		Parametric equations	
Continuity	x	Polar coordinates	
Optimization		Trig manipulations	
		Logs/exponentials	
		Epsilon-delta	
		Listening/reading comprehension	
		Definitions/notation	x

V. CONCLUSION

This particular statics problem depends more strongly on calculus skills than on calculus concepts, similar to the result by Czocher et. al. for the analysis of differential equations. This seems to be in conflict with the general call for greater conceptual math knowledge by engineering faculty. A fuller analysis will uncover whether this phenomenon permeates all of statics and what might explain the discrepancy.

In this problem, the conceptual knowledge of integration isnt needed. Here, integration is only a relationship between the physical quantities $V(x)$ and $M(x)$; the conceptual knowledge that would be involved in constructing this relationship is not required. The nature of integrals as antiderivatives or as sums of many small pieces does not arise in this problem. Students might use the area nature of integrals to evaluate them graphically. However, students often make mistakes while solving problems, and conceptual knowledge of integration might help them identify errors in their solution.

Much like the epistemological mismatches found between calculus and differential equations [1], we found preliminary evidence for epistemological mismatches between how calculus is taught and tested compared to how it is used in statics. This preliminary result suggests that, as in differential equations, continuity is used as a constraint on the solution as opposed to a property to be checked about a function. Continuity is a way to find the proper boundary conditions separating two regimes of behavior to ensure desirable physical qualities.

The full investigation of the alignment between calculus and statics could reveal more mismatches between the two subjects. In addition, there may be some topics (such as the rigorous definition of the limit) that do not occur at all in the following course. With a fully documented list of dependencies for both skills and concepts, engineering departments will be able to provide more precise requests and recommendations to the mathematics departments at their institutions. This technique could be applied in other bottle-neck prerequisite chains such as those in computer science programming sequences, but this would require careful research detailing the concepts and skills from those prerequisite courses as has already been done for calculus.

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